Advances in Efficient Probabilistic Reasoning with Answer Set Semantics

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Der Wissenschaftsfonds.

LOGICAL METHODS IN COMPUTER SCIENCE

Probabilistic Stream Reasoning

Interest in quantitative reasoning on top of Stream Reasoning:

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- Probabilistic Reasoning [Nickles and Mileo, 2014]
- wLARS [Eiter and Kiesel, 2020] for
  - Probabilistic Reasoning
  - Preferential Reasoning
  - Algebraic Model Counting

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  ↪ Our recent work

Approach

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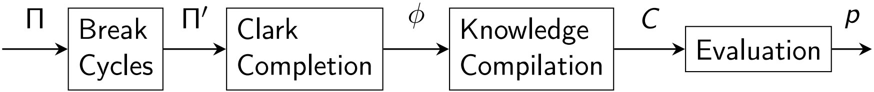
- General strategy:



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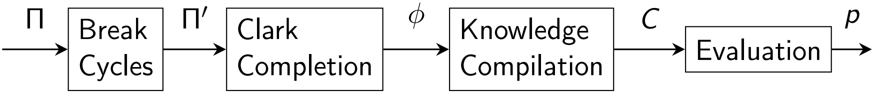

- **Goal:** Perform cycle breaking and Clark completion in such a way that compilation and evaluation are fast

Treewidth I

Definition (Tree decomposition, Treewidth)

Let G be a graph. Then a tree decomposition is a pair (T, χ), where T is a tree and χ is a labeling of $V(T)$ by subsets of $V(G)$ s.t.

- *for all nodes $v \in V(G)$ there is $t \in V(T)$ s.t. $v \in \chi(t)$;*
- *for every edge $\{v_1, v_2\} \in V(E)$ there exists $t \in V(T)$ s.t. $v_1, v_2 \in \chi(t)$;*
- *for all nodes $v \in V(G)$ the set of nodes $\{t \in V(T) \mid v \in \chi(t)\}$ forms a (connected) subtree of T.*

The width of (T, χ) is $\max_{t \in V'} |\chi(t)| - 1$. The treewidth of a graph is the minimal width of any of its tree decompositions.

Treewidth II

CNF:

$$a \vee b$$
$$\neg b \vee c \vee d$$
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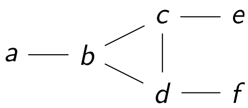

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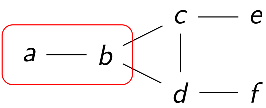

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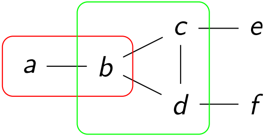

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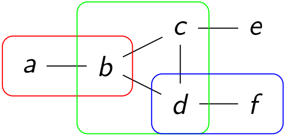

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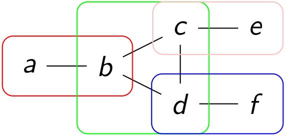

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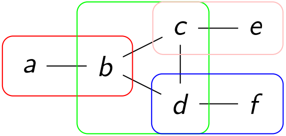
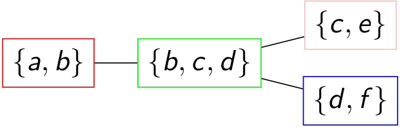


Tree Decomposition:



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- Tree decomposition-based variable selection performs well [Korhonen and Järvisalo, 2021]

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  - ProbLog [Mantadelis and Janssens, 2010]:
    Resulting treewidth is $\mathcal{O}(k \cdot 2^{|C|})$

where $|C|$ is the size of the largest strongly connected component (SCC) of the dependency graph of the program

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- **Idea:** Split the strongly connected components into subgraphs of low cyclicity

Component-Boosted Backdoor Size

Definition ($\mathrm{cbs}(G)$)

Let G be a digraph. Then the component-boosted backdoor size of G, denoted $\mathrm{cbs}(G)$, is

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Intuitively, $\mathrm{cbs}(G)$ measures the *cyclicity* of G by decomposition into "easy to solve" subgraphs

Main Result

Theorem

For every answer set program Π, there exists an equivalent program Π' such that

1. *the answer sets are preserved bijectively*
2. *Π' is tight/acyclic*
3. *the treewidth of Π' is less or equal to $k \cdot \mathrm{cbs}(\mathsf{DEP}(\Pi))$, where k is the treewidth of Π.*

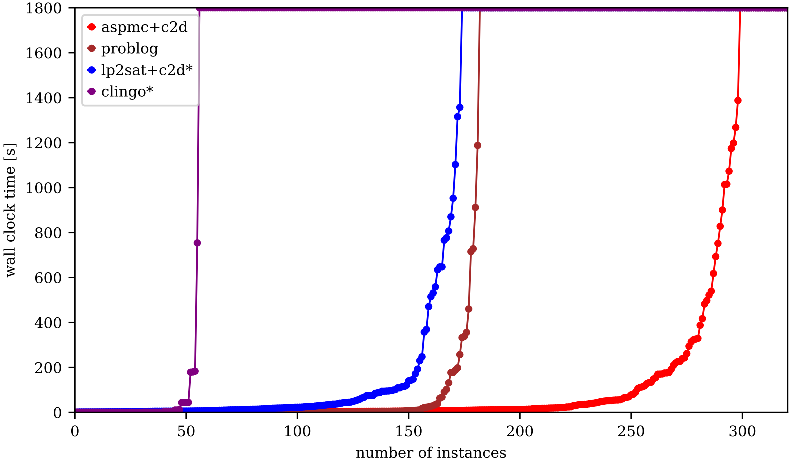
Scenarios

- S1 Probabilistic reasoning: Computing probabilities for atoms of Problog programs
- S2 Counting (small number of solutions on average): Counting the number of different paths between stations in public transport networks
- S3 Counting (many solutions on average): Counting conflict-free extensions in abstract argumentation

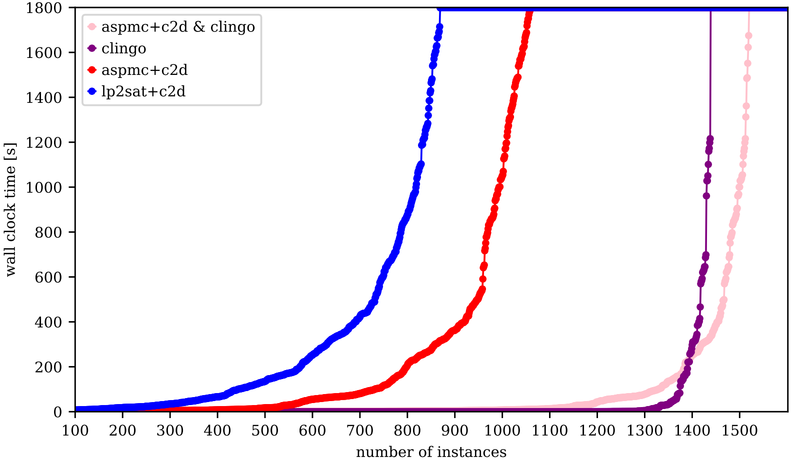
Solvers

- Problog, version 2.1.0.42, run with arguments "-k sdd"
- clingo, version 5.4.0, run with arguments "-q -n 0"
- lp2sat+c2d: cycle breaking due to [Bomanson, 2017] followed by compilation using c2d [Darwiche, 2004]
- aspmc+c2d: our cycle breaking followed by compilation using c2d [Darwiche, 2004]

Results S1


aspmc+c2d
problog
lp2sat+c2d*
clingo*
wall clock time [s]
number of instances
0
200
400
600
800
1000
1200
1400
1600
1800
0
50
100
150
200
250
300

Results S2


aspmc+c2d & clingo
clingo
aspmc+c2d
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300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500

Results S3

solver configuration	$\sum$	tw ranges 0-300	300-600	>600	unique	time[h]
aspmc+c2d	**241**	**185**	**26**	**30**	**12**	**45.16**
lp2sat+c2d	182	182	0	0	0	73.85
clingo	144	97	21	26	2	94.78

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- Improved approach for probabilistic reasoning under ASP semantics
- Need to tackle the time domain!
  $\hookrightarrow$ Compile once and reuse: great!
  $\hookrightarrow$ Already hard for one timepoint, how problematic for more?

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