Weighted LARS for Quantitative Stream Reasoning

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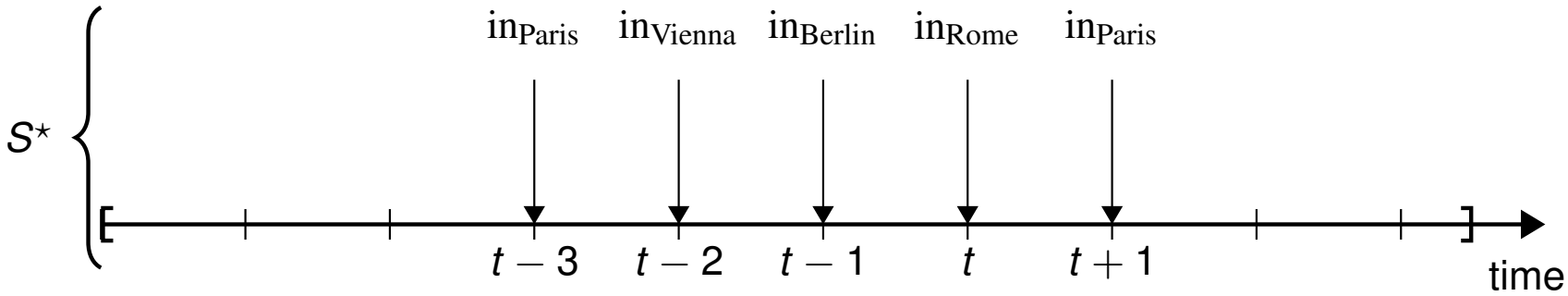
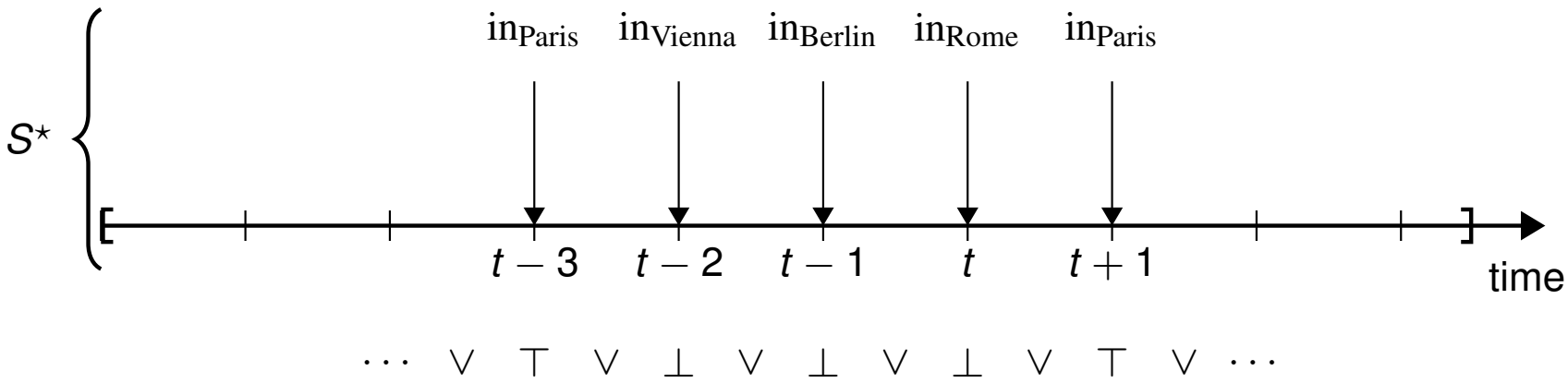
logics LOGICAL METHODS IN COMPUTER SCIENCE

(Qualitative) Stream Reasoning with LARS



- Does Peter visit Paris?

(Qualitative) Stream Reasoning with LARS



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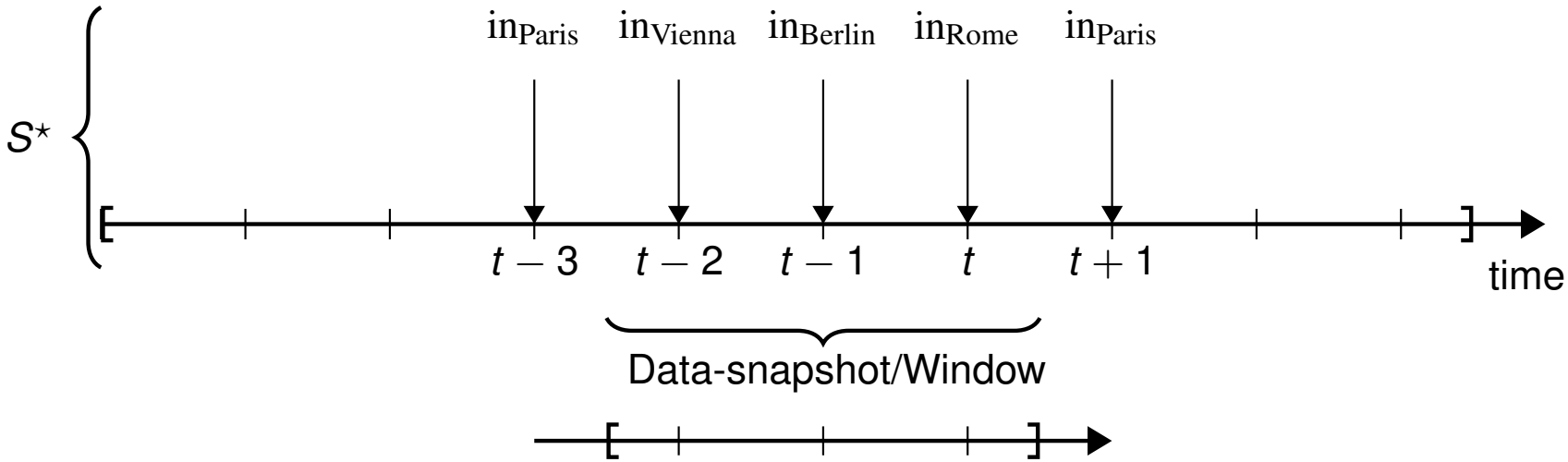

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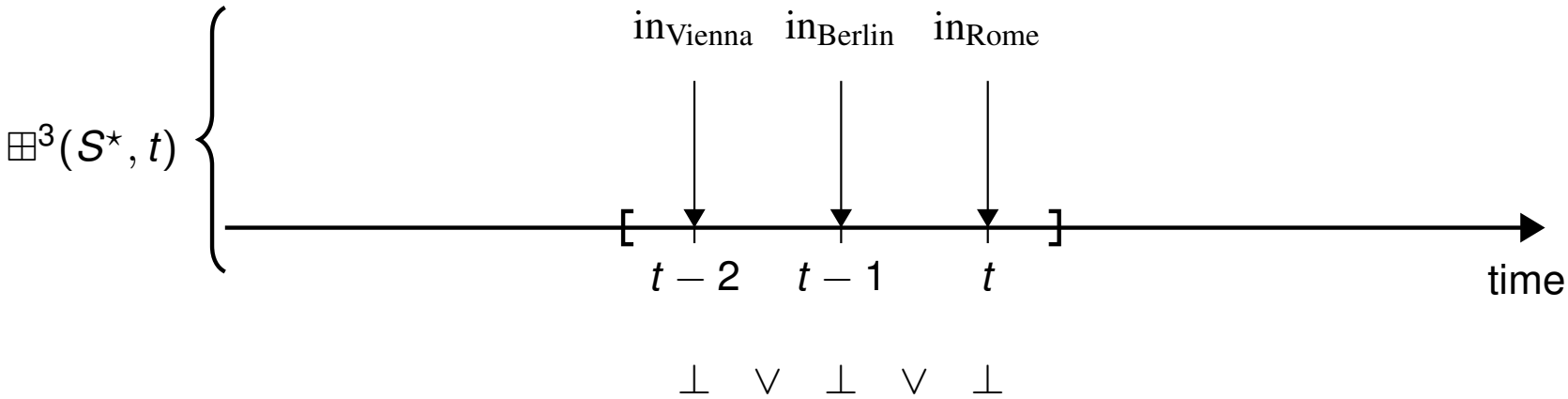

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- Other quantitative questions (Probabilities, Weighted Model Counting, ...)

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- Weighted Logic!
- *Overload* $(\{\bot, \top\}, \vee, \wedge, \bot, \top)$ using semirings $(R, \oplus, \otimes, e_\oplus, e_\otimes)$ and allow semiring values in formulas

Semiring Semantics

Examples are

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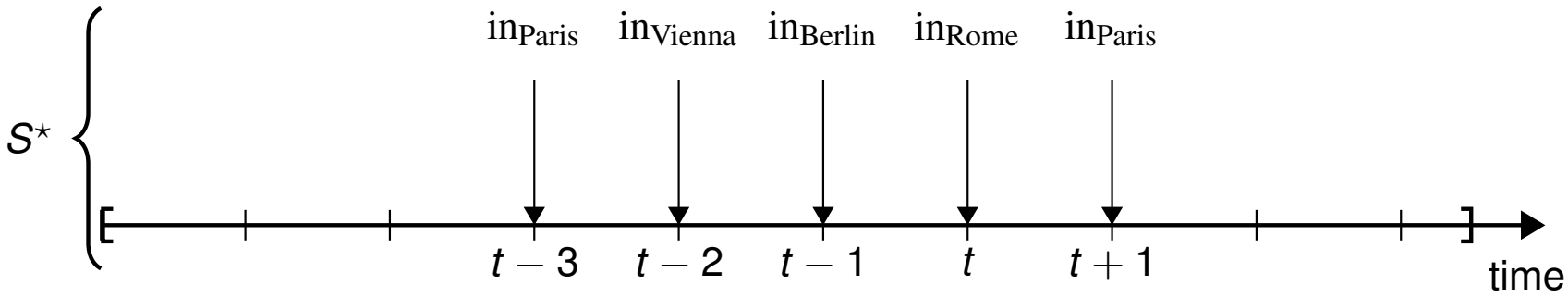
- $\mathcal{R}_{\max} = (\mathbb{Q} \cup \{-\infty, \infty\}, \max, +, -\infty, 0)$, the max-tropical semiring.
  $\hookrightarrow$ disjunction is maximum, conjunction is sum

Example II



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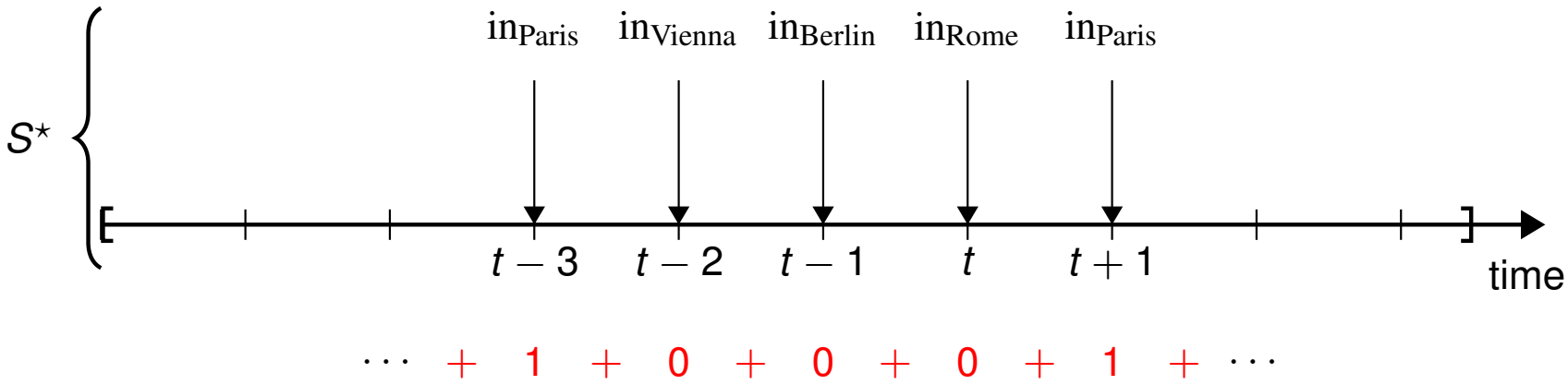

- How often does Peter visit Paris?

$\rightarrow$ $\Diamond\text{in}_{\text{Paris}}$ over the natural number semiring $(\mathbb{N}, +, \cdot, 0, 1)$

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$$\mu(S,t) = \begin{cases} [\![\alpha]\!]_{\mathcal{R}}(S,S,t) & \text{if } S \text{ is an answer stream of } \Pi \text{ at } t, \\ e_{\oplus} & \text{otherwise.} \end{cases}$$

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- S is an answer stream of Π at t if (S,S,t) satisfies Π and (S,S,t) is a minimal model of the reduct $\Pi^{S,t} = \{\alpha \leftarrow \beta \in \Pi \mid (S,S,t) \text{ satisfies } \beta\}$

Expressivity Results I

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- **Weighted Model Counting** by

  computing the sum using $\oplus$
  of $\mu(S, t)$ for each answer streams (S, t)

Expressivity Results I

LARS measures enable subsumption of corresponding ASP-extensions

- Preferential Reasoning, i.e.,

  Weak Constraints [Buccafurri *et al.*, 2000]
  (part.) *asprin* [Brewka *et al.*, 2015]

- Probabilistic Reasoning, i.e.,

  P-log [Baral *et al.*, 2009]
  LP^{MLN} [Lee and Yang, 2017]
  ProbLog [De Raedt *et al.*, 2007]

- Weighted Model Counting, i.e.,

  aProbLog [Kimmig *et al.*, 2011]

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  - Rational Expressions (Regular Expressions with weights)
- Shows the expressiveness of LARS measures
- Gives a rule-based alternative for specification via automata

Complexity Results

The evaluation of LARS measures

- is PSPACE-hard for any non-trivial semiring (LARS is already PSPACE-complete)
- possible in FPSPACE(poly) for under mild restrictions on the semiring and weighted formula

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Preferential Reasoning (over restricted LARS measures)

- Preference Checking is Π_2^p-complete
- Brave Preferential Reasoning is Σ_3^p-complete

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- Next up
  - Implementation
  - Application in object detection, traffic regulation


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Weighted LARS

- Evaluate formulas w.r.t. $(S^\star, S, t)$
- $S^\star$ starting stream, S current stream, t time

Formula	LARS	Weighted LARS
constant	$\bot, \top$	semiring value k
p	true, false	one, zero
$\neg\alpha$	true $\leftrightarrow$ false	zero $\rightarrow$ one, rest $\rightarrow$ zero
$\alpha \wedge \beta$	α and β	α times β
$\alpha \vee \beta$	α or β	α plus β
$\Box\alpha$	for all t: α	product of α over t
$\Diamond\alpha$	exists t: α	sum of α over t
$@_{t'}\alpha$	$(S^\star, S, t)$ changes to $(S^\star, S, t')$	
$\boxplus^w\alpha$	$(S^\star, S, t)$ changes to $(S^\star, w(S, t), t)$	
$\triangleright\alpha$	$(S^\star, S, t)$ changes to $(S^\star, S^\star, t)$	