Quantitative and Stream Extensions of ASP

Rafael Kiesel

Supervisor: Thomas Eiter

22nd of September 2021

Der Wissenschaftsfonds.

LOGICAL METHODS IN COMPUTER SCIENCE

Answer Set Programming

Answer Set Programming (ASP):

- Non-monotonic
- Default Negation

$$a \leftarrow b_1, \ldots, b_n, \text{not } b_{n+1}, \ldots, \text{not } b_m$$

- Solve NP-hard Problems

Extensions

- ASP

$$\text{fail} \leftarrow \text{overheat}$$

Extensions

- ASP

$$\text{fail} \leftarrow \text{overheat}$$

(TD) Temporal Domain (e.g. LARS [Beck *et al.*, 2018]):

$$\text{fail} \leftarrow \boxplus^{5}\Box\text{overheat}$$

Extensions

- ASP

$$\text{fail} \leftarrow \text{overheat}$$

(TD) Temporal Domain (e.g. LARS [Beck *et al.*, 2018]):

$$\text{fail} \leftarrow \boxplus^5 \Box \text{overheat}$$

(QM) Quantitative Reasoning over Models (e.g. *asprin* [Brewka *et al.*, 2015]):

$$\#\text{optimize}(\text{temp})$$

Extensions

- ASP

$$\text{fail} \leftarrow \text{overheat}$$

(TD) Temporal Domain (e.g. LARS [Beck *et al.*, 2018]):

$$\text{fail} \leftarrow \boxplus^5 \Box \text{overheat}$$

(QM) Quantitative Reasoning over Models (e.g. *asprin* [Brewka *et al.*, 2015]):

$$\#\text{optimize}(\text{temp})$$

(QC) Quantitative Constraints (e.g. Weight Constraints [Niemela *et al.*, 1999]):

$$\text{fail} \leftarrow \text{temp}(X), X > 100$$

Problem Statement

Goal

Find and **analyze** a general framework that combines

succinct specifications (QC)
reasoning over answer sets (QM)
temporal domain (TD)

Problem Statement

Goal

Find and **analyze** a general framework that combines

succinct specifications (QC)
reasoning over answer sets (QM)
temporal domain (TD)

Problem Statement

Goal

Find and **analyze** a general framework that combines

succinct specifications (QC)
reasoning over answer sets (QM)
temporal domain (TD)

Problem Statement

Goal

Find and **analyze** a general framework that combines

succinct specifications (QC)
reasoning over answer sets (QM)
temporal domain (TD)

Challenges

Challenge I: No One Fits All

- No single extension that captures all possibilities

Challenges

Challenge I: No One Fits All

- No single extension that captures all possibilities
- Both for QM and QC

Challenges

Challenge I: No One Fits All

- No single extension that captures all possibilities
- Both for QM and QC
- Sometimes even multiple extensions for the same problem

Challenges

Challenge I: No One Fits All

- No single extension that captures all possibilities
- Both for QM and QC
- Sometimes even multiple extensions for the same problem

Challenge II: Entangle Time & Quantitative Reasoning

- Time domain and specification of quantities not orthogonal

Challenges

Challenge I: No One Fits All

- No single extension that captures all possibilities
- Both for QM and QC
- Sometimes even multiple extensions for the same problem

Challenge II: Entangle Time & Quantitative Reasoning

- Time domain and specification of quantities not orthogonal
- Differentiate aggregates at a given timepoint and aggregates over all timepoints

Challenges

Challenge I: No One Fits All

- No single extension that captures all possibilities
- Both for QM and QC
- Sometimes even multiple extensions for the same problem

Challenge II: Entangle Time & Quantitative Reasoning

- Time domain and specification of quantities not orthogonal
- Differentiate aggregates at a given timepoint and aggregates over all timepoints
- Statements of the form $w : \phi$ insufficient!

State of the Art

Quantitative Constraints:

- Hybrid ASP [Cabalar *et al.*, 2020]
- Nested Expressions [Ferraris, 2011]

State of the Art

Quantitative Constraints:

- Hybrid ASP [Cabalar *et al.*, 2020]
- Nested Expressions [Ferraris, 2011]

Quantitative Reasoning over Models:

- LP^{MLN} [Lee and Yang, 2017]
- Algebraic Prolog [Kimmig *et al.*, 2011; Belle and De Raedt, 2016]

State of the Art

Quantitative Constraints:

- Hybrid ASP [Cabalar *et al.*, 2020]
- Nested Expressions [Ferraris, 2011]

Quantitative Reasoning over Models:

- LP^{MLN} [Lee and Yang, 2017]
- Algebraic Prolog [Kimmig *et al.*, 2011; Belle and De Raedt, 2016]

Combination:

- telingo [Cabalar *et al.*, 2018]

Approach: Semirings

- Semirings can be used to parameterise calculations [Green *et al.*, 2007][Bistarelli *et al.*, 2018]

Approach: Semirings

- Semirings can be used to parameterise calculations [Green *et al.*, 2007][Bistarelli *et al.*, 2018]
- A semiring is an algebraic structure $(R, \oplus, \otimes, e_\oplus, e_\otimes)$, s.t.
  - $(R, \oplus, e_\oplus)$ is a commutative monoid with neutral element $e_\oplus$
  - $(R, \otimes, e_\otimes)$ is a monoid with neutral element $e_\otimes$
  - multiplication ($\otimes$) distributes over addition ($\oplus$)
  - multiplication by $e_\oplus$ annihilates R $(\forall r \in R : e_\oplus \otimes r = e_\oplus = r \otimes e_\oplus)$

Semiring Examples

Prominent examples are

- $\mathbb{Q} = (\mathbb{Q}, +, \cdot, 0, 1)$ rational numbers
- $\mathcal{R}_{\max} = (\mathbb{R} \cup \{-\infty\}, \max, +, -\infty, 0)$ max-plus
- $\mathcal{R}_{\min} = (\mathbb{R} \cup \{\infty\}, \min, +, \infty, 0)$ min-plus
- $\mathbb{B} = (\{\bot, \top\}, \vee, \wedge, \bot, \top)$ boolean
- $\mathcal{P}(A) = (\mathcal{P}(A), \cup, \cap, \emptyset, A)$ powerset

Weighted Logic [Droste and Gastin, 2007]

$$\alpha ::= k \mid p \mid \neg p \mid \alpha \wedge \alpha \mid \alpha \vee \alpha \mid \ldots$$

k semiring value, *p* atomic formula

Weighted Logic [Droste and Gastin, 2007]

$$\alpha ::= k \mid p \mid \neg p \mid \alpha * \alpha \mid \alpha + \alpha \mid \ldots$$

k semiring value, *p* atomic formula

Weighted Logic [Droste and Gastin, 2007]

$$\alpha ::= k \mid p \mid \neg p \mid \alpha * \alpha \mid \alpha + \alpha \mid \dots$$

k semiring value, p atomic formula

"Disjunction is addition and conjunction is multiplication"

$$\begin{aligned}
[\![k]\!]_{\mathcal{R}}(\mathcal{I}) &= k \\
[\![p]\!]_{\mathcal{R}}(\mathcal{I}) &= \begin{cases} e_{\otimes}, & \text{if } p \in \mathcal{I} \\ e_{\oplus}, & \text{otherwise.} \end{cases} \\
[\![\alpha * \beta]\!]_{\mathcal{R}}(\mathcal{I}) &= [\![\alpha]\!]_{\mathcal{R}}(\mathcal{I}) \otimes [\![\beta]\!]_{\mathcal{R}}(\mathcal{I}) \\
[\![\alpha + \beta]\!]_{\mathcal{R}}(\mathcal{I}) &= [\![\alpha]\!]_{\mathcal{R}}(\mathcal{I}) \oplus [\![\beta]\!]_{\mathcal{R}}(\mathcal{I}).
\end{aligned}$$

Example

$$\alpha = 15 * \text{Circus} + 20 * \text{Restaurant}$$
$$\mathcal{I} = \{\text{Circus}\}$$

Example

$$\alpha = 15 * \text{Circus} + 20 * \text{Restaurant}$$
$$\mathcal{I} = \{\text{Circus}\}$$

Over the semiring $\mathbb{Q}$:

$$[\![\alpha]\!]_{\mathbb{Q}}(\mathcal{I}) = 15 \cdot 1 + 20 \cdot 0 = 15.$$

Example

$$\alpha = 15 * \text{Circus} + 20 * \text{Restaurant}$$
$$\mathcal{I} = \{\text{Circus}\}$$

Over the semiring $\mathbb{Q}$:

$$[\![\alpha]\!]_{\mathbb{Q}}(\mathcal{I}) = 15 \cdot 1 + 20 \cdot 0 = 15.$$

Over the min tropical semiring $\mathcal{R}_{\text{min}} = (\mathbb{R} \cup \{\infty\}, \min, +, \infty, 0)$:

$$[\![\alpha]\!]_{\mathcal{R}_{\text{min}}}(\mathcal{I}) = \min(15 + 0, 20 + \infty) = 15.$$

Appeal of Weighted Logic

- Recall the principle:
  "Disjunction is addition and conjunction is multiplication"

Appeal of Weighted Logic

- Recall the principle:

  "Disjunction is addition and conjunction is multiplication"
- Semantics often defined via disjunction and conjunction

Appeal of Weighted Logic

- Recall the principle:

  "Disjunction is addition and conjunction is multiplication"
- Semantics often defined via disjunction and conjunction
- E.g. existential quantification over timepoints is sum over timepoints

  $\hookrightarrow$ Weighted Logic as a generic tool

Appeal of Weighted Logic

- Recall the principle:
  "Disjunction is addition and conjunction is multiplication"
- Semantics often defined via disjunction and conjunction
- E.g. existential quantification over timepoints is sum over timepoints
  $\hookrightarrow$ Weighted Logic as a generic tool
- E.g. Here-and-There Logic $\rightarrow$ non-monotonicity
  $\hookrightarrow$ Weighted Here-and-There Logic $\rightarrow$ non-monotonic calculation

Quantitative Reasoning over Models

- Thomas Eiter and Rafael K_, Weighted LARS for Quantitative Stream Reasoning, *European Conference on Artificial Intelligence, 2020*

Quantitative Reasoning over Models

- Thomas Eiter and Rafael K_, Weighted LARS for Quantitative Stream Reasoning, *European Conference on Artificial Intelligence, 2020*
- LARS is a stream reasoning framework with answer set semantics

Quantitative Reasoning over Models

- Thomas Eiter and Rafael K_, Weighted LARS for Quantitative Stream Reasoning, *European Conference on Artificial Intelligence, 2020*
- LARS is a stream reasoning framework with answer set semantics
- Introduced a weighted version of LARS
  $\hookrightarrow$ Addresses challenge II

Quantitative Reasoning over Models

- Thomas Eiter and Rafael K_, Weighted LARS for Quantitative Stream Reasoning, *European Conference on Artificial Intelligence, 2020*
- LARS is a stream reasoning framework with answer set semantics
- Introduced a weighted version of LARS
  $\hookrightarrow$ Addresses challenge II
- Showed its power as an underlying framework for
  - Probabilities
  - Preferences
  - Weighted Model Counting

  $\hookrightarrow$ Addresses challenge I

Quantitative Constraints

- Thomas Eiter and Rafael K_, ASP$\mathcal{AC}$: Answer Set Programming with Algebraic Constraints, *International Conference on Logic Programming, 2020*

Quantitative Constraints

- Thomas Eiter and Rafael K_, ASP$\mathcal{AC}$: Answer Set Programming with Algebraic Constraints, *International Conference on Logic Programming, 2020*
- Introduced First-Order Weighted Here-and-There Logic
  $\hookrightarrow$ Calculations over semirings with non-monotonic dependency

Quantitative Constraints

- Thomas Eiter and Rafael K_, ASP$\mathcal{AC}$: Answer Set Programming with Algebraic Constraints, *International Conference on Logic Programming, 2020*
- Introduced First-Order Weighted Here-and-There Logic
  $\hookrightarrow$ Calculations over semirings with non-monotonic dependency
- Algebraic Constraints subsume
  - Aggregates
  - Choice Constraints
  - Weight Constraints with Conditionals
  - ...

  $\hookrightarrow$ Addresses challenge I

Quantitative Constraints

- Thomas Eiter and Rafael K_, ASP$\mathcal{AC}$: Answer Set Programming with Algebraic Constraints, *International Conference on Logic Programming, 2020*
- Introduced First-Order Weighted Here-and-There Logic
  $\hookrightarrow$ Calculations over semirings with non-monotonic dependency
- Algebraic Constraints subsume (with mild practical restrictions)
  - Aggregates
  - Choice Constraints
  - Weight Constraints with Conditionals
  - ...

  $\hookrightarrow$ Addresses challenge I

Semiring Complexity

- Thomas Eiter and Rafael K_, On the Complexity of Sum-Of-Products Problems over Semirings, *AAAI Conference on Artificial Intelligence, 2021*

Semiring Complexity

- Thomas Eiter and Rafael K_, On the Complexity of Sum-Of-Products Problems over Semirings, *AAAI Conference on Artificial Intelligence, 2021*
- In depth analysis of the complexity of weighted model counting over semirings

Semiring Complexity

- Thomas Eiter and Rafael K_, On the Complexity of Sum-Of-Products Problems over Semirings, *AAAI Conference on Artificial Intelligence, 2021*
- In depth analysis of the complexity of weighted model counting over semirings
- Can be very hard (up to undecidable)

Semiring Complexity

- Thomas Eiter and Rafael K_, On the Complexity of Sum-Of-Products Problems over Semirings, *AAAI Conference on Artificial Intelligence, 2021*
- In depth analysis of the complexity of weighted model counting over semirings
- Can be very hard (up to undecidable)
- We identified classes that can be solved with #SAT or SAT-solvers

Implementation

- Thomas Eiter, Markus Hecher and Rafael K_, Treewidth-Aware Cycle-Breaking for Algebraic Answer Set Counting, *International Conference on Principles of Knowledge Representation and Reasoning, 2021 (to Appear)*

Implementation

- Thomas Eiter, Markus Hecher and Rafael K_, Treewidth-Aware Cycle-Breaking for Algebraic Answer Set Counting, *International Conference on Principles of Knowledge Representation and Reasoning, 2021 (to Appear)*
- Novel *treewidth-aware* translation from ASP to CNF

Implementation

- Thomas Eiter, Markus Hecher and Rafael K_, Treewidth-Aware Cycle-Breaking for Algebraic Answer Set Counting, *International Conference on Principles of Knowledge Representation and Reasoning, 2021 (to Appear)*
- Novel *treewidth-aware* translation from ASP to CNF
- Prototypical implementation: aspmc

Implementation

- Thomas Eiter, Markus Hecher and Rafael K_, Treewidth-Aware Cycle-Breaking for Algebraic Answer Set Counting, *International Conference on Principles of Knowledge Representation and Reasoning, 2021 (to Appear)*
- Novel *treewidth-aware* translation from ASP to CNF
- Prototypical implementation: aspmc
- aspmc partially outperforms other solvers on standard benchmarks

What is next?

- Practical evaluation on real world example

What is next?

- Practical evaluation on real world example
- Additional improvements of the efficiency of aspmc

What is next?

- Practical evaluation on real world example
- Additional improvements of the efficiency of aspmc
- Further analysis and comparisons in journal versions

Summary

- Weighted Logic is to quantities what Here-And-There Logic is to non-monotonicity

Summary

- Weighted Logic is to quantities what Here-And-There Logic is to non-monotonicity
- Unify quantitative extensions

Summary

- Weighted Logic is to quantities what Here-And-There Logic is to non-monotonicity
- Unify quantitative extensions
- Easily adjustable to the given domain

Summary

- Weighted Logic is to quantities what Here-And-There Logic is to non-monotonicity
- Unify quantitative extensions
- Easily adjustable to the given domain
- General formalism is found and partially implemented
  $\hookrightarrow$ further analysis and real world application

Harald Beck, Minh Dao-Tran, and Thomas Eiter.
Lars: A logic-based framework for analytic reasoning over streams.
Artificial Intelligence, 261:16–70, 2018.

Vaishak Belle and Luc De Raedt.
Semiring programming: A framework for search, inference and learning.
arXiv preprint arXiv:1609.06954, 2016.

Stefano Bistarelli, Fabio Rossi, and Francesco Santini.
A novel weighted defence and its relaxation in abstract argumentation.
International Journal of Approximate Reasoning, 92:66–86, 2018.

Gerhard Brewka, James Delgrande, Javier Romero, and Torsten Schaub.
asprin: Customizing answer set preferences without a headache.
In *Twenty-Ninth AAAI Conference on Artificial Intelligence*, 2015.

Pedro Cabalar, Roland Kaminski, Torsten Schaub, and Anna Schuhmann.
Temporal answer set programming on finite traces.
Theory and Practice of Logic Programming, 18(3-4):406–420, 2018.

Pedro Cabalar, Jorge Fandinno, Torsten Schaub, and Philipp Wanko.
A uniform treatment of aggregates and constraints in hybrid ASP.

arXiv preprint arXiv:2003.04176, 2020.

Manfred Droste and Paul Gastin.
Weighted automata and weighted logics.
Theoretical Computer Science, 380(1):69, 2007.

Paolo Ferraris.
Logic programs with propositional connectives and aggregates.
ACM Transactions on Computational Logic (TOCL), 12(4):25, 2011.

Todd J Green, Grigoris Karvounarakis, and Val Tannen.
Provenance semirings.
In *Proceedings of the twenty-sixth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems*, pages 31–40. ACM, 2007.

Angelika Kimmig, Guy Van den Broeck, and Luc De Raedt.
An algebraic prolog for reasoning about possible worlds.
In *Twenty-Fifth AAAI Conference on Artificial Intelligence*, 2011.

Joohyung Lee and Zhun Yang.
Lpmln, weak constraints, and p-log.
In *Thirty-First AAAI Conference on Artificial Intelligence*, 2017.

Ilkka Niemela, Patrik Simons, and Timo Soininen.
Stable model semantics of weight constraint rules.
In *International Conference on Logic Programming and Nonmonotonic Reasoning*, pages 317–331. Springer, 1999.