Reasoning on Multi-Relational Contextual Hierarchies via ASP with Algebraic Measures

[1]Loris Bozzato, [2]Thomas Eiter, [2]Rafael Kiesel

[1]DKM, Fondazione Bruno Kessler – Trento, Italy

[2]Inst. für Informationssysteme, TU Wien – Wien, Austria

partially funded by Humane AI project #820437 and FWF project W1255-N23

23rd of September 2021

FWF Der Wissenschaftsfonds. HUMANE AI NET logics LOGICAL METHODS IN COMPUTER SCIENCE

Context Representation

- Context representation is a well-known area of study in KR
  - [McCarthy, 1993]
  - [Lenat, 1998]
  - [Giunchiglia and Serafini, 1994]
- Semantic Web: need to interpret datasets in correct context
- Different proposals for DL-based representation of contexts
  - [Klarman, 2013]
  - [Brewka and Eiter, 2007]
  - [Bao *et al.*, 2010]
  - [Straccia *et al.*, 2010]

Contextualized Knowledge Repositories (CKR)

DL-based framework for reasoning with contextual knowledge in the semantic web [Bozzato *et al.* 2019,2018b]

Contextualized Knowledge Repositories (CKR)

A CKR $\mathfrak{K} = \langle \mathfrak{C}, K_{\mathsf{N}} \rangle$ consists of

- Global Knowledge $\mathfrak{C} = \langle \mathsf{N}, \succ \rangle$:
  The relation $\succ$ between the different contexts $c \in \mathsf{N}$
- Contextual Knowledge $K_{\mathsf{N}} = (K_c)_{c \in \mathsf{N}}$:
  The *additional* axioms K_c in context $c \in \mathsf{N}$

In local contexts we have, apart from usual DL rules

- Defeasible Axioms $D(C \sqsubseteq D)$:
  Can be overridden in more specific contexts
- Eval-Expressions $eval(c, D)$:
  Can reference the state of D in another context c

CKR Example

c_{world} :
$$E \sqcap R \sqsubseteq \bot$$
$$D(S \sqsubseteq E)$$

↑

c_{branch} :
$$OS \sqcap RE \sqsubseteq \bot$$
$$S \sqsubseteq OS$$
$$D(S \sqsubseteq R)$$

↑

c_{local} : $S(i)$

- *E*lectronics, *R*obotics, *S*upervisor
- *O*n*S*ite, *RE*mote
- At c_{local} we have $S(i), OS(i)$ and $R(i)$
- $S \sqsubseteq OS$ is actually defeasible w.r.t. time!
- $D(S \sqsubseteq R)$ actually only holds since 2020!

Multi-Relational CKR

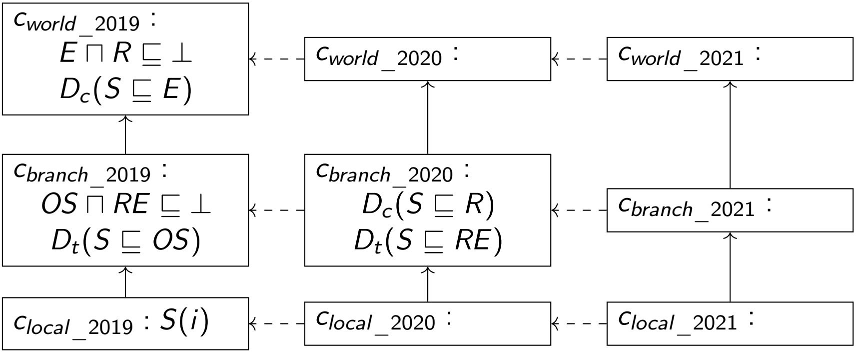
- We need different relations between contexts!
- Axioms may be defeasible *with respect to a relation*!

Multi-Relational CKR (MR-CKR)

- Global Knowledge $\mathfrak{C} = \langle \mathrm{N}, \succ_1, \ldots, \succ_m \rangle$:
  The relations $\succ_i$ between the different contexts $c \in \mathrm{N}$
- Contextual Knowledge $K_\mathrm{N} = (K_c)_{c \in \mathrm{N}}$:
  The *additional* axioms K_c in context $c \in \mathrm{N}$

- Defeasible Axioms $D_i(C \sqsubseteq D)$:
  Can be overridden in more specific contexts w.r.t. $\succ_i$

MR-CKR Example



- $\longrightarrow$ for $\succ_c$, the coverage relation
- $\dashrightarrow$ for $\succ_t$, the time relation
- At c_{local_2019} we have $S(i)$, $OS(i)$ and $E(i)$
- At c_{local_2020} and c_{local_2021} we have $S(i)$, $RE(i)$ and $R(i)$

Semantics I

- Clashing assumption $\langle \alpha, e \rangle$: instance e is an exception of $D_r(\alpha)$
- CAS-interpretation $\mathfrak{I}_{CAS} = \langle \mathcal{I}, \chi_1, \ldots, \chi_m \rangle$:
  - $\mathcal{I}(c)$: interpretation of context c
  - $\chi_i(c)$: set of clashing assumptions of context c w.r.t. relation $\succ_i$

(Justified) CAS-model $\mathfrak{I}_{CAS} \models \mathfrak{K}$

$\mathfrak{I}_{CAS}$ is a CAS-model for $\mathfrak{K}$ if:

1. $\mathcal{I}(c') \models K_c$, if $c' \preceq_* c$
2. for every $D_i(\alpha) \in K_c$ and $c' \preceq_{-i} c$, $\mathcal{I}(c') \models \alpha$
3. for every $D_i(\alpha) \in K_c$ and $c'' \prec_i c' \preceq_{-i} c$, if $\langle \alpha, e \rangle \notin \chi_i(c'')$, then $\mathcal{I}(c'') \models \alpha(e)$

$\mathfrak{I}_{CAS}$ is justified if each clashing assumption $\langle \alpha, e \rangle \in \chi(c)$ is justified by some clashing set S such that

- $\mathcal{I}(c) \models S$
- $S \cup \{\alpha(e)\}$ is unsatisfiable

Semantics II

Which justified CAS-Models are preferred?

(LP) Locally, we prefer those that satisfy more specific defeasible axioms

(RP) On the relation level, we prefer those that have an improvement locally and no change for the worse otherwise

(GP) Globally, we prefer those that are preferred on the relation of the smallest index

Reasoning with MR-CKR

How can we reason with MR-CKR?

- Restrict the DL language to $\mathcal{SROIQ}$-RL and obtain least models via translation to ASP [Bozzato *et al.*, 2018a]

But how can we get only the preferred models?

- Preferences + ASP ?$\rightarrow$ asprin [Brewka *et al.*, 2015]
- asprin only allows strict partial orders but we can have cyclic preference relations $\lightning$

We investigated two options:

1. Restriction to *eval*-disconnected MR CKR: avoid cycles
2. Use algebraic measures [Eiter and Kiesel, 2020]

eval-Disconnectedness

Idea:

- If $\mathfrak{K}$ is *eval*-free, i.e., there are no *eval*-expressions at all, the interpretations $\mathcal{I}(c)$ and $\mathcal{I}(c')$ for $c \neq c'$ are independent
- Then any interpretation $(\mathcal{I}(c))_{c \in \mathsf{N}}$, where $\mathcal{I}(c)$ is locally preferred, is also globally preferred

We want at least some *eval*-expressions though!

eval-Disconnectedness

- Generalizes this idea
- Is a syntactic condition that can be checked easily
- Prevents dependence of the satisfaction of a default at context c on the satisfaction of another default at context c'

asprin Encoding

Local Preference (LP)

#preference(LP(c,i),poset){
  $\neg ovr(\alpha, X, c, i)$ >> $ovr(\alpha, X, c, i)$;
  $\neg ovr(\alpha_2, Y, c, i)$ >> $\neg ovr(\alpha_1, X, c, i)$; for $c_1 \succeq_{-i} c_{1b} \succ_i c$ and
    $c_2 \succeq_{-i} c_{2b} \succ_i c$ and $c_{1b} \succ_i c_{2b}$ and $D_i(\alpha_i)$ in K_{c_i}.}.

Relation-global Preference (RP)

#preference(RP(i),pareto){**LP(C,i) : context(C)}.

GlobalPreference (GP)

```
#preference(GP,lexico){W::**RP(I) : rel_w(I,W)}.
```

Correctness

Putting things together:

- $PK(\mathfrak{K})$ is the answer set program that encodes the MR-CKR $\mathfrak{K}$
- P_{pref} is the preference encoding in asprin including `#optimize(GP)`.

Theorem

Let $\mathfrak{K}$ be a multi-relational CKR that is eval-disconnected and in $\mathcal{SROIQ}$-RLD normal form. Then under the unique name assumption,

1. *for every α and c such that $O(\alpha, c)$ is defined, $\mathfrak{K} \models c : \alpha$ iff $PK(\mathfrak{K}) \cup P_{pref} \models O(\alpha, c)$;*
2. *for every BCQ $Q = \exists y.\gamma(y)$ on $\mathfrak{K}$, $\mathfrak{K} \models Q$ iff $PK(\mathfrak{K}) \cup P_{pref} \models O(Q)$.*

We can use $PK(\mathfrak{K}) \cup P_{pref}$ to reduce reasoning tasks to ASP+asprin!
$\hookrightarrow$ implemented in the publicly available tool CKRew!

Weighted Logic [Droste and Gastin, 2007]

- Use weighted formulas α over a semiring $(R, \oplus, \otimes, e_\oplus, e_\otimes)$ of the form

$$\alpha ::= k \mid p \mid \neg p \mid \alpha + \alpha \mid \alpha * \alpha,$$

  where $k \in R$ and p is a propositional variable
- Allows calculations over a semiring depending on the truth of propositional variables or formulas
- Example: $2 * \mathit{candy} + 3 * \mathit{pasta}$ over semiring $(\mathbb{N}, +, \cdot, 0, 1)$
  $\hookrightarrow$ if we buy *candy* and *pasta* we pay 5

Algebraic Measures

- An Algebraic Measure μ is defined by a triple $\langle \Pi, \alpha, \mathcal{R} \rangle$, where
  - Π is an ASP program
  - α is a weighted LARS formula over $\mathcal{R}$
  - $\mathcal{R}$ is a semiring
- The weight of an answer set $S \in \mathcal{AS}(\Pi)$ is

$$\mu(S) = [\![\alpha]\!]_{\mathcal{R}}(S).$$

- Intuitively, algebraic measures allow us to associate a weight with an answer set
  $\hookrightarrow$ many possibilities what to do with this weight!
- The overall weight of μ is defined as

$$\mu(\Pi) = \bigoplus_{S \in \mathcal{AS}(\Pi)} \mu(S).$$

Preference Encoding

Preferred Answer Sets

An answer set $S \in \mathcal{AS}(\Pi)$ is preferred w.r.t. a measure $\mu = \langle \Pi, \alpha, \mathcal{R} \rangle$ and a relation $>$ on R if no $S' \in \mathcal{AS}(\Pi)$ exists such that $\mu(S') > \mu(S)$

How do can we use this?

- The powerset semiring $\mathcal{P}(CA)$ over the set of possible clashing assumption can do "bookkeeping"
- Define a weighted formula that checks for clashing assumptions $\alpha = \Sigma_{\langle \phi, e, c, i \rangle \in CA} ovr(\phi, e, c, i) * \{\langle \phi, e, c, i \rangle\}$.
- Take the relation $>_{opt}$ on the semiring values $S \subseteq CA$ that correctly captures the preference on the justified models.
- $S \in \mathcal{AS}(PK(\mathfrak{K}))$ is preferred iff it corresponds to a least preferred CAS model $\langle \mathfrak{I}, \overline{\chi} \rangle$ of $\mathfrak{K}$

Additional Possibilities with Overall Weight Queries

For single-relational, *eval*-free CKR $\mathfrak{K}$, we can also use overall weight queries fruitfully

Theorem

There exists a measure $\mu_{one} = \langle PK(\mathfrak{K}), \alpha_{one}, \mathcal{R}_{one}(\mathfrak{K})\rangle$ *whose overall weight is either*

- *the minimum lexicographical preferred answer set of* $PK(\mathfrak{K})$
- *or* 0 *if there is no preferred answer set.*

Theorem

There exists a measure $\mu_{all} = \langle PK(\mathfrak{K}), \alpha_{all}, \mathcal{R}_{all}(\mathfrak{K})\rangle$ *whose overall weight characterizes all preferred answer sets.*

Conclusions

- Multi-Relational CKR allows us to properly capture differences in contexts w.r.t. different dimensions
- CKRew: Implementation using an encoding in ASP + asprin for *eval*-disconnected MR-CKR
- Algebraic Measures as a general to approach many quantitative problems
  $\hookrightarrow$ probabilistic reasoning
  $\hookrightarrow$ preferential reasoning
  $\hookrightarrow$ parameter learning
  $\hookrightarrow$ and more

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