ICT-48-2020: Towards a vibrant European Network of AI excellence centres

Reasoning on Contextual Hierarchies via ASP with Algebraic Measures

L. Bozzato[1], T. Eiter[2], R. Kiesel[2], L. Serafini[1], P. Schüller[2]

[1]DKM, Fondazione Bruno Kessler – Trento, Italy

[2]Knowledge-Based Systems Group, Vienna University of Technology (TU Wien) – Wien, Austria

January 2021 - May 2021

Context Representation

- Context representation is a well-known area of study in KR
- Semantic Web: need to interpret datasets in correct context
- Different proposals for DL-based representation of contexts

Contextualized Knowledge Repositories (CKR)

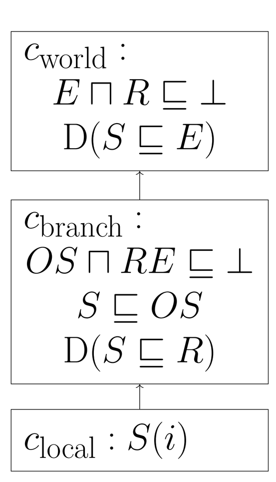
DL-based framework for reasoning with contextual knowledge in the semantic web [2]
A CKR $\mathfrak{K} = \langle \mathfrak{C}, \mathrm{K}_{\mathbf{N}} \rangle$ consists of

- Global Knowledge $\mathfrak{C} = \langle \mathbf{N}, \succ \rangle$:
  The relation $\succ$ between the different contexts $c \in \mathbf{N}$
- Contextual Knowledge $\mathrm{K}_{\mathbf{N}} = (K_c)_{c \in \mathbf{N}}$:
  The *additional* axioms K_c in context $c \in \mathbf{N}$

In local contexts we have, apart from usual DL rules

- Defeasible Axioms $D(C \sqsubseteq D)$:
  Can be overridden in more specific contexts
- Eval-Expressions $eval(c, D)$:
  Can reference the state of D in another context c

CKR Example

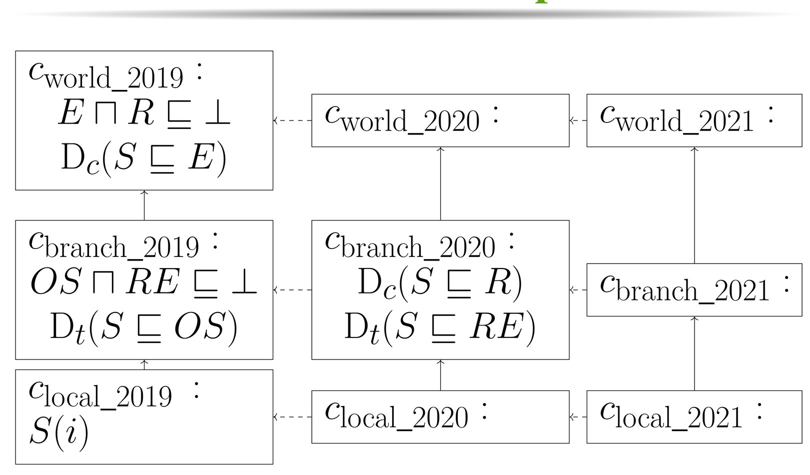


- *E*lectronics, *R*obotics, *S*upervisor
- *On**S*ite, *RE*mote
- At c_{local} we have $S(i), OS(i)$ and $R(i)$
- $S \sqsubseteq OS$ is actually defeasible w.r.t. time!
- $\mathrm{D}(S \sqsubseteq R)$ actually only holds since 2020!

Multi-Relational CKR

- We need different relations between contexts!
- Axioms may be defeasible *with respect to a relation*!
- Global Knowledge $\mathfrak{C} = \langle \mathbf{N}, \succ_1, \ldots, \succ_m \rangle$:
  The relations $\succ_i$ between the different contexts $c \in \mathbf{N}$
- Contextual Knowledge $\mathrm{K}_{\mathbf{N}} = (K_c)_{c \in \mathbf{N}}$:
  The *additional* axioms K_c in context $c \in \mathbf{N}$
- Defeasible Axioms $D_i(C \sqsubseteq D)$:
  Can be overridden in more specific contexts w.r.t. $\succ_i$

MR-CKR Example



- → for $\succ_c$, the coverage relation
- ⇢ for $\succ_t$, the time relation
- At $c_{\text{local\_2019}}$ we have $S(i), OS(i)$ and $E(i)$
- At $c_{\text{local\_2020}}$ and $c_{\text{local\_2021}}$ we have $S(i), RE(i)$ and $R(i)$

Semantics

- Clashing assumption $\langle \alpha, \mathbf{e} \rangle$: instance $\mathbf{e}$ is an exception of $\mathrm{D}_r(\alpha)$
- CAS-interpretation $\mathfrak{I}_{CAS} = \langle \mathcal{I}, \chi_1, \ldots, \chi_m \rangle$:
  - $\mathcal{I}(c)$: interpretation of context c
  - $\chi_i(c)$: set of clashing assumptions of context c w.r.t. relation $\succ_i$

(Justified) CAS-model $\mathfrak{I}_{CAS} \models \mathfrak{K}$

$\mathfrak{I}_{CAS}$ is a CAS-model for $\mathfrak{K}$ if:

1. $\mathcal{I}(\mathsf{c}') \models \mathrm{K}_{\mathsf{c}}$, if $\mathsf{c}' \preceq_* \mathsf{c}$
2. for every $\mathrm{D}_i(\alpha) \in \mathrm{K}_{\mathsf{c}}$ and $\mathsf{c}' \preceq_{-i} \mathsf{c}$, $\mathcal{I}(\mathsf{c}') \models \alpha$
3. for every $\mathrm{D}_i(\alpha) \in \mathrm{K}_{\mathsf{c}}$ and $\mathsf{c}'' \prec_i \mathsf{c}' \preceq_{-i} \mathsf{c}$, if $\langle \alpha, \mathbf{e} \rangle \notin \chi_i(\mathsf{c}'')$, then $\mathcal{I}(\mathsf{c}'') \models \alpha(\mathbf{e})$

$\mathfrak{I}_{CAS}$ is justified if each clashing assumption $\langle \alpha, \mathbf{e} \rangle \in \chi(\mathsf{c})$ is justified by some clashing set S such that

- $\mathcal{I}(c) \models S$
- $S \cup \{\alpha(\mathbf{e})\}$ is unsatisfiable

Which justified CAS-Models are preferred?

(LP) Locally, we prefer those that satisfy more specific defeasible axioms: $\chi_i(\mathsf{c}) > \chi'_i(\mathsf{c})$, if
- for every $\eta = \langle \alpha, \mathbf{e} \rangle \in \chi_i(\mathsf{c}) \setminus \chi'_i(\mathsf{c})$ with $\mathrm{D}_i(\alpha)$ at context $\mathsf{c}_1 \succeq_{-i} \mathsf{c}_{1b} \succ_i \mathsf{c}$,
- there exists $\eta' = \langle \alpha', \mathbf{f} \rangle \in \chi'_i(\mathsf{c}) \setminus \chi_i(\mathsf{c})$ with $\mathrm{D}(\alpha')$ at context $\mathsf{c}_2 \succeq_{-i} \mathsf{c}_{2b} \succ_i \mathsf{c}$
- such that $\mathsf{c}_{1b} \succ_i \mathsf{c}_{2b}$

(RP) On the relation level, we prefer those that have an improvement locally and no change for the worse otherwise: $\chi_i > \chi'_i$, if
- there exists $\mathsf{c} \in \mathbf{N}$ s.t. $\chi_i(\mathsf{c}) > \chi'_i(\mathsf{c})$ and not $\chi'_i(\mathsf{c}) > \chi_i(\mathsf{c})$
- for no context $\mathsf{c}' \neq \mathsf{c} \in \mathbf{N}$ it holds that $\chi_i(\mathsf{c}') < \chi'_i(\mathsf{c}')$ and not $\chi'_i(\mathsf{c}') < \chi_i(\mathsf{c}')$.

(GP) Globally, we prefer those that are preferred on the relation of the smallest index: $\langle \mathcal{I}, \chi_1, \ldots, \chi_m \rangle > \langle \mathcal{I}', \chi'_1, \ldots, \chi'_m \rangle$, if
- there exists $i \in \{1, \ldots, m\}$ such that $\chi_i > \chi'_i$
- for all $j < i \in \{1, \ldots, m\}$ it holds that $\chi_j \not< \chi'_j$

Reasoning with MR-CKR

How can we reason with MR-CKR?

- Restrict the DL language to $\mathcal{SROIQ}$-RL and obtain least models via translation to ASP [1]

But how can we get only the preferred models?

- Preferences + ASP → asprin [3]
- asprin only allows strict partial orders but we can have cyclic preference relations ↯

We investigated two options:

1. Restriction to *eval*-disconnected MR CKR: avoid cycles
2. Use algebraic measures

eval-Disconnectedness

Idea:

- If $\mathfrak{K}$ is *eval*-free, i.e., there are no *eval*-expressions at all, the interpretations $\mathcal{I}(c)$ and $\mathcal{I}(c')$ for $c \neq c'$ are independent
- Then any interpretation $(\mathcal{I}(c))_{c \in \mathbf{N}}$, where $\mathcal{I}(c)$ is locally preferred, is also globally preferred

We want at least some *eval*-expressions though!
Introduce *eval*-Disconnectedness

- Generalizes this idea
- Is a syntactic condition that can be checked easily
- Prevents dependence of the satisfaction of a default at context c on the satisfaction of another default at context c'

asprin Encoding

Local Preference (LP)

```
#preference(LP(c,i),poset){
  ¬ovr(α,X,c,i) >> ovr(α,X,c,i);
  ¬ovr(α2,Y,c,i) >> ¬ovr(α1,X,c,i);
   for c1 ⪰−i c1b ≻i c and
   c2 ⪰−i c2b ≻i c and c1b ≻i c2b and
   Di(αi) in Kci. }.
```

Relation-global Preference (RP)

```
#preference(RP(i),pareto){
  **LP(C,i) : context(C) }.
```

GlobalPreference (GP)

```
#preference(GP,lexico){
  W::**RP(I) : rel_w(I,W) }.
```

Correctness

Putting things together:

- $PK(\mathfrak{K})$ is the answer set program that encodes the MR-CKR $\mathfrak{K}$
- P_{pref} is the preference encoding in asprin including `#optimize(GP)`.

Theorem

Let $\mathfrak{K}$ be a multi-relational CKR that is eval-disconnected and in $\mathcal{SROIQ}$-RLD normal form. Then under the unique name assumption,

1. *for every α and c such that $O(\alpha, \mathsf{c})$ is defined, $\mathfrak{K} \models \mathsf{c} : \alpha$ iff $PK(\mathfrak{K}) \cup P_{pref} \models O(\alpha, \mathsf{c})$;*
2. *for every BCQ $Q = \exists \mathbf{y}.\gamma(\mathbf{y})$ on $\mathfrak{K}$, $\mathfrak{K} \models Q$ iff $PK(\mathfrak{K}) \cup P_{pref} \models O(Q)$.*

We can use $PK(\mathfrak{K}) \cup P_{pref}$ to reduce reasoning tasks to ASP+asprin!
↪ implemented in the publicly available tool CKRew[a]!

[a] Paper and Implementation

Paper: "Reasoning on Multi-Relational Contextual Hierarchies via Answer Set Programming with Algebraic Measures," ICLP 2021.
Prototype & Tech. Report: available at `https://www.ai4europe.eu/research/ai-catalog/ckrew-ckr-datalog-rewriter`

Weighted Logic [4]

- Use weighted formulas α over a semiring $(R, \oplus, \otimes, e_\oplus, e_\otimes)$ of the form
  $$\alpha ::= k \mid p \mid \neg p \mid \alpha + \alpha \mid \alpha * \alpha,$$
  where $k \in R$ and p is a propositional variable
- Allows calculations over a semiring depending on the truth of propositional variables or formulas
- Example: $2 * \text{candy} + 3 * \text{pasta}$ over semiring $(\mathbb{N}, +, \cdot, 0, 1)$
  ↪ if we buy candy and pasta we pay 5

Algebraic Measures

- An Algebraic Measure μ is defined by a triple $\langle \Pi, \alpha, \mathcal{R} \rangle$, where
  - Π is an ASP program
  - α is a weighted LARS formula over $\mathcal{R}$
  - $\mathcal{R}$ is a semiring
- The weight of an answer set $S \in \mathcal{AS}(\Pi)$ is
  $$\mu(S) = [\![\alpha]\!]_{\mathcal{R}}(S).$$
- Intuitively, algebraic measures allow us to associate a weight with an answer set
  ↪ many possibilities what to do with this weight!
- The overall weight of μ is defined as
  $$\mu(\Pi) = \bigoplus\nolimits_{S \in \mathcal{AS}(\Pi)} \mu(S).$$
- An answer set $S \in \mathcal{AS}(\Pi)$ is preferred w.r.t. a measure $\mu = \langle \Pi, \alpha, \mathcal{R} \rangle$ and a relation $>$ on R if no $S' \in \mathcal{AS}(\Pi)$ exists such that $\mu(S') > \mu(S)$

Preference Encoding

We can use this as follows

- The powerset semiring $\mathcal{P}(CA)$ over the set of possible clashing assumption can do "bookkeeping"
- Define a weighted formula that checks for clashing assumptions
  $\alpha = \Sigma_{\langle \phi, \mathbf{e}, \mathsf{c}, i \rangle \in CA} \mathtt{ovr}(\phi, \mathbf{e}, \mathsf{c}, i) * \{\langle \phi, \mathbf{e}, \mathsf{c}, i \rangle\}$.
- Take the relation $>_{opt}$ on the semiring values $S \subseteq CA$ that correctly captures the preference on the justified models.
- $S \in \mathcal{AS}(PK(\mathfrak{K}))$ is preferred iff it corresponds to a least preferred CAS model $\langle \mathfrak{I}, \overline{\chi} \rangle$ of $\mathfrak{K}$

Conclusions

- Multi-Relational CKR allows us to properly capture differences in contexts w.r.t. different dimensions
- CKRew: Implementation using an encoding in ASP + asprin for *eval*-disconnected MR-CKR
- Algebraic Measures as a general to approach many quantitative problems
  ↪ probabilistic reasoning
  ↪ preferential reasoning
  ↪ parameter learning
  ↪ and more

References

[1] Loris Bozzato, Thomas Eiter, and Luciano Serafini. Enhancing context knowledge repositories with justifiable exceptions. *Artif. Intell.*, 257:72–126, 2018.

[2] Loris Bozzato, Thomas Eiter, and Luciano Serafini. Justifiable exceptions in general contextual hierarchies. In Gábor Bella and Paolo Bouquet, editors, *Modeling and Using Context. CONTEXT 2019*, volume 11939 of *Lecture Notes in Computer Science*, pages 26–39. Springer, 2019.

[3] Gerhard Brewka, James Delgrande, Javier Romero, and Torsten Schaub. asprin: Customizing answer set preferences without a headache. In *Twenty-Ninth AAAI Conference on Artificial Intelligence*, 2015.

[4] Manfred Droste and Paul Gastin. Weighted automata and weighted logics. *Theoretical Computer Science*, 380(1):69, 2007.